Sanjay Ghodawat University, Kolhapur 2017-18

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FY B Sc	**School of Science**	**Semester I**
MTS 101	**Mathematics I**	**Max Marks: 100**
~~**Nov 2017**~~ 2 DEC 2017	**End Semester Examination (ESE)**	**Time: 3 Hrs**

Instructions for Students: 1) Use of non-programmable calculator is allowed

2) All questions are compulsory

Que.No	Questions	CO	Marks
Que.1	Select the correct alternative (two marks each)	**All CO**	**16**
	A) $\lim_{x\to 0} x\, sin\left(\frac{1}{x}\right)$= ________. i) 0 iii) 1 ii) 2 iv) ∞	**CO1**	**2**
	B) $\lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x-1}\right)$ = ________. i) 0 iii) 1 ii) $\frac{1}{2}$ iv) ∞	**CO1**	**2**
	C) Consider the statements I) Every differentiable function is continuous. II) Every continuous function is differentiable. Then________. i) Only I is true iii) Both I and II are true. ii) Only II is true iv) Both I and II are false.	**CO2**	**2**
	D) If $y = 10^{100x}$ then y_{100} = ______. i) $10^{100}\, 10^{x}$ iii) $100^{100} 10^{100x} (\log 10)^{100}$ ii) $10^{10} x^{10} (\log 10)^{10}$ iv) $10^{100} 100^{10x} (\log 10)^{10}$	**CO3**	**2**
	E) The infinite series $1 + x + x^2 + x^3 + \cdots$ is the expansion of ______. i) $\log(1 + x)$ iii) e^{-x} ii) e^{x} iv) $(1 - x)^{-1}$	**CO4**	**2**

CO5 2

F) The value of $\lim_{x\to 0}\frac{(1-cosx)(1-e^{x})}{2x^{3}}$ is______.

i) 2 iii) ½
ii) -1/4 iv) ¼

G) Which of the following is true? CO6 2

i) $cosh^2x + sinh^2x = 1$ iii) $e^{-x} = cosx - i\,sinx$
ii) $sinix = sinhx$ iv) $\cosh\left(\frac{\pi}{3}\right)i = 1/2$

H) The value of $\left(sin\frac{\pi}{6} + i\,cos\frac{\pi}{6}\right)^6 =$ ______. CO6 2

i) i iii) -1
ii) $-i$ iv) 1

Que.2 Attempt any **Two** of the following: (seven marks each) CO1 14

CO1 7

a) Define continuous function. Hence show that the addition and multiplication of continuous function is continuous separately.

b) Discuss the continuity of $f(x)$ at $x = 0$, where CO1 7

$$f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} \qquad if\ x \neq 0$$

$$= 0 \qquad if\ x = 0$$

c) Find α and β if the function $f(x)$ is continuous in (-2,2), where CO1 7

$$f(x) = \begin{cases} x + \alpha, & if -2 < x < 0 \\ 2x + 1, & if\ 0 \leq x < 1 \\ \beta - x, & if\ 1 \leq x < 2 \end{cases}$$

Que.3 Attempt any **Two** of the following: (seven marks each) CO2 14

a) State and prove Cauchy's mean value theorem. CO2 7

CO2 7

b) Prove that between any two real roots of the equation $e^x sinx = 1$, there is at least one real root of $e^x cosx + 1 = 0$.

CO2 7

c) Prove that $0 < \frac{1}{x}\log\left(\frac{e^x - 1}{x}\right) < 1$.

Que.4		Attempt any **Two** of the following: (seven marks each)	CO3	14
	a)	State and prove the Leibnitz's theorem.	CO3	7
	b)	If $y = e^{msin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$	CO3	7
	c)	If $y = a\cos(\log x) + bsin(\log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$	CO3	7
Que.5		Attempt any **Two** of the following: (seven marks each)	CO4	14
	a)	State and prove Taylor's theorem for expansion of functions.	CO4	7
	b)	Expand e^{sinx} by Maclaurin's theorem.	CO4	7
	c)	Expand $tan^{-1}x$ in powers of $\left(x - \frac{\pi}{4}\right)$.	CO4	7
Que.6		Attempt any **Two** of the following: (seven marks each)	CO5	14
	a)	State and prove L' Hospital's Rule.	CO5	7
	b)	Evaluate $\lim_{x\to 0} \frac{tanx - x}{x^2 tanx}$	CO5	7
	c)	Evaluate $\lim_{x\to 0} \frac{\sqrt{(1+x^2)} - \sqrt{(1-x^2)}}{x^2}$	CO5	7
Que.7		Attempt any **Two** of the following: (seven marks each)	CO6	14
	a)	State and prove De Moivres theorem.	CO6	7
	b)	If α and β are the roots of the equation $x^2 - 2x + 2 = 0$ then Prove that $\alpha^n + \beta^n = 2^{\left(\frac{n+2}{2}\right)}\cos\left(\frac{n\pi}{4}\right)$.	CO6	7
	c)	If $\tan(u+iv) = x+iy$ then show that i) $x^2 + y^2 + 2xcot2u = 1$ ii) $x^2 + y^2 - 2ycoth2v + 1 = 0$	CO6	7
Total				10(