



# Sanjay Ghodawat University, Kolhapur

2017-18

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

FY M.Sc

School of Science

Semester I

MTS 503

Real Analysis

Max Marks: 100

7 Dec 2017

End Semester Examination (ESE)

Time: 3 Hrs

- Instructions for Students:** 1) Use of non-programmable calculator is allowed.  
2) All questions are compulsory.

Q.1)	Attempt the following: (two marks each)	12 Marks	CO
	a) The Sequence $\{1/n\}$ is (i) Bounded & Convergent (ii) Unbounded & convergent (iii) Bounded & divergent (iv) Divergent & unbounded.	2	CO4
	b) A metric space X is Compact (i) It is sequentially compact (ii) It is sequentially bounded (iii) It does not satisfies both (iv) none.	2	CO3
	c) The set of Real number R (i) Finite set (ii) Bounded set (iii) Closed set (iv) None.	2	CO3
	d) The series $1+3+5+7\dots$ (i) Unbounded (ii) Convergent (iii) Divergent (iv) None.	2	CO4
	e) Cardinal Number is (i) Finite set (ii) infinite set (iii) Void set (iv) empty set.	2	CO1
	f) Denumerable set is (i) Countable (ii) Uncountable (iii) Infinite (iv) None.	2	CO1
Q.2)	Attempt any two of the following: (ten marks each)	20 Marks	CO
	a) State and prove contraction mapping theorem.	10	CO2
	b) State and prove completeness property of R.	10	CO1
	c) State and prove Arzela –Ascoli Theorem.	10	CO4

Q.3)	Attempt any two of the following: (eight marks each)	16 Marks	CO
	a) State and prove stone-weierstrass theorem.	08	CO4
	b) State and prove fundamental theorem of calculus.	08	CO5
	c) State and prove Archimedian property.	08	CO1
Q.4)	Attempt any two of the following: (six marks each)	12 Marks	CO
	a) Define convergent sequence. If $\{a_n\}$ is convergent rational sequence then $\langle a_n \rangle$ is bounded.	06	CO4
	b) Define ordred field and it's axioms with an example.	06	CO1
	c) Define linear transformation and prove that if $A \in L(\mathbb{R}^n, \mathbb{R}^m)$ then norm of A is less than infinity and A is uniformly continuous mapping of $(\mathbb{R}^n, \mathbb{R}^m)$	06	CO2
Q.5)	Attempt any three of the following: (four marks each)	12 Marks	CO
	a) Suppose $\{f_n\}$ is a sequence of function differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point $x_0$ on $[a,b]$ . If $\{f_n'\}$ converges uniformly on closed interval $[a,b]$ to a function $f'$ $\{f_n'\}$ converges to $f'$ and $f'(x) = \lim_{n \rightarrow \infty} f_n'(x)$ .	04	CO2
	b) Define power series? Find the radius of convergence of power series. $\sum_{n=1}^{\infty} z^n.$	04	C04
	c) Define the existence of the Riemann integral with upper and lower Riemann integral and give one example Which is not Reimann integrable?	04	CO5
	d) Show that there does not exist rational number $r$ , $r^2 = 2$ .	04	CO1

Q.6)	Attempt any four of the following: (seven marks each)	28 Marks	CO
	a) State and prove law of trichotomy.	07	CO1
	b) Define the existence of the Lebesgue integral and prove following results. Let $f$ & $g$ be bounded measurable function on a set $E$ then	07	CO5
	(i) $\int_E a f = a \int_E f$		
	(ii) $\int_E (f+g) = \int_E f + \int_E g$		
	c) If $K$ is compact metric space if $f_n \in F(K)$ For $n=1,2,3..$ and if sequence $\{f_n\}$ converges uniformly on $K$ then sequence $\{f_n\}$ is eqicontinuous on $K$ .	07	CO4
	d) If $\langle a_n \rangle$ & $\langle b_n \rangle \in F_Q$ then prove the following results	07	CO1
	(i) $\langle a_n + b_n \rangle \in F_Q$ (ii) $\langle a_n - b_n \rangle \in F_Q$ (iii) $\langle a_n / b_n \rangle \in F_Q$ , where $\lim_{n \rightarrow \infty} b_n \neq 0$ .		
	e) Prove that every cauchy sequence of rationals is bounded.	07	CO4