Sanjay Ghodawat University, Kolhapur 2017-18

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FY M Sc	**School of Sciences | Department of Mathematics**	**Semester I**
MTS 501	**Linear Algebra**	**Max Marks: 100**
5 Dec 2017	**End Semester Examination**	**Time: 3 Hrs.**

Instructions for Students: 1) Use of non-programmable calculator is allowed
2) All questions are compulsory

Q.1 Choose correct alternative for the following questions (12) — Marks, CO

a) *Let A and B be non-empty subsets of a vector space V. $A \subseteq B$, Then* — 2, CO1

A) If B is LI then so is A.

B) If B is LD then so is A.

C) If A is LI then so is B.

D) If B is generating set, then so is A.

b) *If V and U be vector space of dimension 4 and 6 respectively. Then $\dim Hom(V,U)$ is* — 2, CO2

A) 4   B) 6   C) 10   D) 24

c) *Let V be the inner product space consisting of linear polynomials,* $p:[0\ \ 1]\to R$ *(i.e V consists of polynomials p of the form* $p(x)=ax+b,\ a,\in R$*), with the inner product defined by* $\langle p\ \ q\rangle=\int_0^1 p(x)\,q(x)\,dx$ *for* $p,q\in V$*. An orthonormal basis of V is* — 2 — CO5

$A)\{1,\ x\}\quad B)\{1,\ x\sqrt{3}\}\quad C)\{1,\ (2x-1)\sqrt{3}\}\quad D)\{1,x-\frac{1}{2}\}$

d) If $T:V\to V$ is a linear transformation for $\dim(V)=n$ and T has n distinct eigen values then — 2 — CO3

A) T must be invertible

B) T must be diagonalizable

C) T must be invertible as well as diagonalizable

D) T is not diagonalizable

e) *The dimension of the vector space of all symmetric matrics of order* $n\times n$ $(n\geq 2)$ *with real entries and trace equal to zero is* — 2 — CO1

$A)\{(n^2-n)/2\}-1\qquad B)\{(n^2+n)/2\}-1$

$C)\{(n^2-2n)/2\}-1\qquad D)\{(n^2+2n)/2\}-1$

f) *Suppose the matrix* $A=\begin{bmatrix}40 & -29 & -11\\ -18 & 30 & -12\\ 26 & 24 & -50\end{bmatrix}$ *has a certain complex numbers* $\lambda\neq 0$ *as an eigen values.* — 2 — CO3

which of the following numbers must also be an eigen value of A

$A)\lambda+20\qquad B)\lambda-20\qquad C)20-\lambda\qquad D)-20-\lambda$

Q.2		Attempt any **two** of the following. (20 marks)		
	a)	*Show that if* V *is a F.D.V.S. over* F *and* W *is a subspace of* V *then,* $\dim W + \dim A(W) = \dim V$	10	CO2
	b)	*Prove that the set of all linear transformation from a vector* V *into* V *denoted by* $Hom(V,V)$ *is an algebra over* F	10	CO3
	c)	*If* $T \in A(V)$ *then show that* $T^* \in A(V)$, *Moreover* *i)* $(T^*)^* = T$ *ii)* $(S+T)^* = S^* + T^*$ *iii)* $(\lambda S)^* = \bar{\lambda} S^*$ *iv)* $(ST)^* = T^* S^*$	10	CO4
Q.3		Attempt any **two** of the following (16 marks)		
	a)	*Let* V *be a finite dimensional vector space over the field* F. *Let* W *be a subspace of* V. *Then show that* $\dim\left(\frac{V}{W}\right) = \dim(V) - \dim(W)$.	8	CO2
	b)	*If* $\lambda \in F$ *is a characteristic root of* $T \in A(V)$ *where* V *is FDVS. Then show that* λ *is a root of minimal polynomial of* T. *In perticular,* T *has only a finite number of characteristic root in* F.	8	CO3
	c)	Define bilinear form. *Let* V *be a vector space over the field* F *and* L_1, L_2 *be linear functional on* V. *If* $f: V \times V \to F$ *is defined by* $f(\alpha,\beta) = L_1(\alpha)L_2(\beta)$ *then show that* f *is a bilinear form on* V	8	CO5

Q.4		Attempt any **Two** of the following. (12 marks)		
	a)	Show that if W is a subspace of a vector space $V(F)$ then $L(W)=W$ and conversely.	6	CO1
	b)	*Show that if* $A,B\in F_n$ *then* $\det(AB)=(\det A)(\det B)$	6	CO4
	c)	*Define invariant. If* $V(F)$ *is a vector space and* $T:V\to V$ *is any LT on* V*, then show that the range of* T *and null space of* T *and both invariant under* T.	6	CO3
Q.5		Attempt any **Three** of the following. (12 marks)		
	a)	*Define Transpose. Show that if* $A,B\in F_n$ *then* *i)* $(A')'=A$ *ii)* $(A+B)'=A'+B'$	4	CO4
	b)	*If* V *is a vector space over* F *and* $T:V\to V$ *is a linear transformation on* V*. Suppose* f *is a bilinear form on* V *then show that* g *is a bilinear form from* $V\times V$ *into* F *defined as* $g(\alpha,\beta)=f(T(\alpha),T(\beta))$	4	CO5
	c)	*Let* W *be the Vector space of all real, polynomials of degree at most 3. Define* $T:W\to W$ *by* $T(p(x))=p'(x)$ *where* p' *is the derivative of* p*. Then find the matrix of* T *in the basis* $\{1,x,x^2,x^3\}$	4	CO2
	d)	Define Linear Span. Show that $L(S)$ is smallest subspace of V containing S	4	CO1
Q.6		Attempt any **Four** of the following. (28 marks)		
	a)	*Show that* $T\geq 0$ *iff* $T=AA^*$ *for some* A	7	CO4

b)	*Show that if* $T \in A(V)$ *has all it's characteristic roots in* F. *Then there is a basis of* V *in which the matrix of* T *is triangular*	7	CO3
c)	*Define matrix of* f *in bilinear form.* Pr*ove that any* n dim*ensional vector space* V *over* F *and any ordered basis* β *for* V, $\psi_{\beta} : B(V) \rightarrow M_{n\times n}(F)$ *is an isomorphim.*	7	CO5
d)	Pr*ove that if* V *is an* $n-$dim*ensional vector space over the field* F. *and* $B = \{\alpha_1, \cdots \alpha_n\}$ *is a basis of* V. *Then there is uniquely* det*er*min*d basis* $B' = \{f_1, \cdots, f_n\}$ *for* $\widehat{V}$ *s.t* $f_i(\alpha_j) = \partial_{ij}$	7	CO2
e)	Show that minimal generating set of vector space V is a basis of V and conversely every basis of V is a minimal generating set of V.	7	CO1