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Sanjay Ghodawat University, Kolhapur 2017-18

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

FY M Sc.
MTS 509
DEC 2017

School of Science
Ordinary Differential Equations
End Semester Examination (ESE)

Semester I
Max Marks: 100
Time: 3 Hrs.

Instructions for Students: 1) Use of non-programmable calculator is allowed
2) All questions are compulsory

			Marks	COs
Q1		Choose correct alternative for the following questions		
	a)	If $p(r)=(r-r_1)^2$ is a characteristic polynomial then $\phi_1(x)=\dots\dots\dots \& \phi_2(x)=\dots\dots\dots$ are solutions of the differential equation $y''-2r_1y'+r_1^2y=0$ 1. $x^2 \& x^{-3}$ 2. $e^{r_1x} \& e^{r_2x}$ 3. $x^{r_1x} \& x^{r_1x}\log x$ 4. $e^{r_1x} \& xe^{r_1x}$	2	CO1
	b)	If $\phi_1,\phi_2,\dots,\phi_n$ are linearly independent functions on an interval I then any subset of them forms a ....set of functions on I 1. Linearly independent 2. linearly dependent 3. Null 4. None of these	2	CO1
	c)	For a differential equation $L(y)=y''+\frac{y'}{x}-\frac{y}{x^2}=0$, $\phi_1(x)=\dots\dots\dots \& \phi_2(x)=\dots\dots\dots$ are solutions 1. x^2, x^{-3} 2. x^2, x 3. x, x^{-1} 4. x, x^2	2	CO2
	d)	The general solution of $x^2y''+2xy'-6y=0$ is $\phi(x)=\dots\dots\dots$ 1. $c_1x^2+c_2x^{-2}$ 2. $c_1x^2+c_2x^{-3}$ 3. $c_1x^{-2}+c_2x^3$ 4. $c_1e^{-x}+c_2e^{2x}$	2	CO3
	e)	An upper bound of M for $f(x,y)=x^2+y^2$ on $R:\lvert x\rvert\le 1,\lvert y\rvert\le 1$ is 1. 1 2. 3 3. 4 4. 2	2	CO4

f) The Eigen values of a regular Sturm Liouville BVP are………… 2 CO5

1. Imaginary
2. Real
3. Infinite
4. Zero

Q2 Attempt any two of the following

a) Let ϕ_1 and ϕ_2 be two different functions on an interval I. Which are not necessarily solutions of an equation $L(y)=0$. Prove the following 10 CO1

I. If ϕ_1 and ϕ_2 are linearly dependent on I then $W(\phi_1,\phi_2)(x)=0$ for all x in I

II. If $W(\phi_1,\phi_2)(x_0)\neq 0$ for some x_0 in I then ϕ_1 and ϕ_2 are linearly independent on I

III. $W(\phi_1,\phi_2)(x)=0$ for all x in I does not imply that ϕ_1 and ϕ_2 are linearly dependent on I

IV. $W(\phi_1,\phi_2)(x)=0$ for all x in I and $\phi_2(x)\neq 0$ on I, Imply that are ϕ_1 and ϕ_2 are linearly dependent

b) Let $b_1, b_2, b_3, \dots b_n$ be non negative constants such that for all x in I $|a_i(x)|\le b_i,\quad i=1,2,3,\dots,n$ and define k by $k=1+b_1+b_2+b_3+\dots+b_n$ 10 CO2

If x_0 be a point in I & ϕ is a solution of $L(y)=y^{(n)}+a_1(x)y^{(n-1)}+\dots+a_n(x)y=0$ on an interval I. Then for all x in I., $\|\phi(x_0)\|e^{-k|x-x_0|}\le\|\phi(x)\|\le\|\phi(x_0)\|e^{k|x-x_0|}$.

c) Find all the solutions of the form $\phi(x)=|x|^r\sum_{k=0}^{\infty}c_k x^k \ (|x|>0)$ for the equation $3x^2y''+5xy'+3xy=0$ 10 CO3

Q3 Attempt any two of the following

a) If $\phi_1(x)$ is a solution of $L(y)=y''+a_1(x)y'+a_2(x)y=0 \text{ on } I$ and $\phi_1(x)\neq 0$ on I, the second solution $\phi_2(x)$ is given by 8 CO2

$$\phi_2(x)=\phi_1(x)\int_{x_0}^{x}\frac{1}{\left[\phi_1(s)\right]^2}e^{-\int_{x_0}^{s}a_1(t)dt}ds$$

b) A function ϕ is a solution of the initial value problem $y'=f(x,y),\ y(x_0)=y_0$ on an interval I if and only if it is a solution of the integral solution of the integral solution 8 CO4

$$y=y_0+\int_{x_0}^{x}f(t,y)dt \text{ on } I$$

c) Let u and v be two Eigen functions of Sturm-Liouville BVP — 8 — CO5

$$\frac{d}{dx}\left[p(x)\frac{du}{dx}\right]+q(x)u+\lambda r(x)u=0, a\le x\le b$$

$$\alpha_1 u(a)+\beta_1 u'(a)=0 \text{..........................}(2.1)$$

$$\alpha_2 u(b)+\beta_2 u'(b)=0 \text{..........................}(2.2)$$

Corresponding to Eigen values λ and μ then

$$[pW(u.v)]_a^b=0 \text{..................}(2.3)$$

Where W(u,v) is the wornskian of u and v.
If p(a)=0 then (2.3) holds without use of (2.1)
If p(b)=0 then (2.3) holds without use of (2.2)

Q4 Attempt any two of the following

a) Find all the solutions of differential equation $y''-y'=x$ — 6 — CO1

b) Find all solutions of equation for x>0 , $x^2y''+xy'+4y=1$ — 6 — CO3

c) By computing appropriate Lipschitz constants show that the following functions satisfy Lipschitz conditions on the set S — 6 — CO4

1. $f(x,y)=4x^2+y^2, on\, S=\{(x,y)/|x|\le 1, |y|\le 1\}$
2. $f(x,y)=x^2\cos^2 y+y\sin^2 x, on\, S=\{(x,y)/|x|\le 1, |y|<\infty\}$

Q5 Attempt any three of the following

a) Consider the equation $y''+\frac{1}{x}y'-\frac{1}{x^2y}=0 \;\; for\, x>0$ — 4 — CO2

Find two solutions for x>0 and prove that they are linearly independent

b) Classify the singular points in the finite plane of the equation — 4 — CO3

$$x^4(x^2+1)(x-1)^2y''+4x^3(x-1)y'+(x+1)y=0$$

c) Suppose S is either a rectangle $|x-x_0|\le a$, $|y-y_0|\le b$ (a,b>0) or a strip $|x-x_0|\le a$, $|y|\le\infty$ (a>0). And that f is a real valued function defined on S. Such that ∂f/∂y exists and continuous on S and — 4 — CO4

$$\left|\frac{\partial f(x,y)}{\partial y}\right|\le K, for\,(x,y)\in S\;\&\; for\, some\, K>0.$$

d) Show that the eigen values of Sturm-Liouville BVP — 4 — CO5

$u''(x)+\lambda u(x)=0, 0\le x\le L$ with conditions $u(0)=0, \& u(L)=0$ are all positive

Q6 Attempt any four of the following

a) If ϕ_1, ϕ_2 are two solutions of $L(y)=0$ on an interval I containing a point x_0 then $W(\phi_1,\phi_2)(x)=e^{-a_1(x-x_0)}W(\phi_1,\phi_2)(x_0)$ — 7 — CO1

b)	One solution of $x^2y''-2y=0$ on $0<x<\infty$ is $\phi_1(x)=x^2$. Find all the solutions of $x^2y''-2y=2x-1$ on $0<x<\infty$	7	CO2
c)	Consider the equation of order n $L(y)=x^n y^{(n)}+a_1x^{n-1}y^{(n-1)}+....+a_ny=0$ where $a_1,a_2,...,a_n$ are constants. Let $r_1,r_2,....,r_s$ ne distinct roots of the indicial polynomial $q(r)=r(r-1)(r-2)...(r-n+1)+a_1r(r-1)(r-2)...(r-n+2)+....+a_n$ and suppose r_i has multiplicity m_i. Then n functions $\lvert x\rvert^{r_1},\lvert x\rvert^{r_1}\log\lvert x\rvert,\lvert x\rvert^{r_1}(\log\lvert x\rvert)^2,...,\lvert x\rvert^{r_1}(\log\lvert x\rvert)^{m_1-1}$, $\lvert x\rvert^{r_2},\lvert x\rvert^{r_2}\log\lvert x\rvert,\lvert x\rvert^{r_2}(\log\lvert x\rvert)^2,...,\lvert x\rvert^{r_2}(\log\lvert x\rvert)^{m_2-1},...,$ $\lvert x\rvert^{r_s},\lvert x\rvert^{r_s}\log\lvert x\rvert,\lvert x\rvert^{r_s}(\log\lvert x\rvert)^2,...,\lvert x\rvert^{r_s}(\log\lvert x\rvert)^{m_s-1}$ form a basis for the solution of $L(y)=0$ on any interval not containing zero	7	CO3
d)	For the following problem compute the first four approximations $\phi_0, \phi_1, \phi_2, \phi_3$, $y'=x^2+y^2, y(0)=0$	7	CO4
e)	Integrate the following differential equations by constructing Green's functions $y''+10y'+25y=\sin x$	7	CO5

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