



# Sanjay Ghodawat University, Kolhapur

2017-18

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FY M.Sc

School of Science

Semester I

MTS 505

Topology

Max Marks: 100

9 Dec 2017

End Semester Examination (ESE)

Time: 3 Hrs

Instructions for Students: 1) Use of non-programmable calculator is allowed  
2) All questions are compulsory

Q.1)	Attempt the following: (two marks each)	12 Marks	CO
	i) The complement of any open set is (a) Empty set (b) Closed set (c) Open set (d) None.	02	CO1
	ii) The Sierpinski space $(X,S)$ is (a) Connected space (b) Path connected space (c) Disconnected space (d) None.	02	CO4
	iii) $\emptyset$ is empty set then $cl(\emptyset)$ is (a) Open in $R$ (b) Closed in $R$ (c) Dense in $R$ (d) Both (a) and (b)	02	CO2
	iv) Relative topology is known as (a) Induced topology (b) Subspace topology (c) Both (a) & (b) (d) none	02	CO2
	v) A Compact subset of a metric space is (a) Closed (b) open (c) unbounded (d) none.	02	CO4
	vi) $(R,U)$ is (a) Connected (b) Locally connected (c) Both (a) and (b) (d) none.	02	CO5
Q.2)	Attempt any two of the following: (ten marks each)	20 Marks	CO
	a) Define interior subset of a topological space $X$ with an example. Let $X$ is any topological space. Let $A$ and $B$ be any two subsets of $X$ then prove the followings: (i) $X^\circ = X$ . (ii) $\emptyset^\circ = \emptyset$ . (iii) $A^\circ \subseteq B^\circ$ If $A \subseteq B$ .	10	CO1

	<p>b) Define Basis and Sub-basis for topological space with an example. Let <math>(X, T)</math> be a topological space and <math>B \subseteq T</math> then prove that following statements are equivalent.</p> <p>(i) <math>B</math> is a basis for <math>T</math></p> <p>(ii) For each <math>G \in T</math> and <math>x \in G</math> there exists a member <math>U \in B</math> such that <math>x \in U \subseteq G</math>.</p> <p>c) Define Kuratowski closure operator and prove the results of Kuratowski closure operator.</p>	10	CO2
		10	CO2
Q.3)	Attempt any two of the following: (eight marks each)	16 Marks	CO
	<p>(a) Define derived point of a topological space with an example. Let <math>(X, T)</math> be a topological space and <math>A</math> be any subset of <math>X</math> then <math>x \in \text{cl}(A)</math> if and only if <math>G \cap A \neq \emptyset</math>.</p> <p>(b) Define closed map of a topological space. If a function <math>f: X \rightarrow Y</math> is closed map then show that <math>\forall S \subseteq Y</math> and <math>\forall</math> open set <math>f^{-1}(S) \subseteq U \exists</math> an open set <math>S \subseteq V \ni f^{-1}(V) \subseteq U</math>.</p> <p>(c) Define Neighbourhood of point with an example. Prove that each member <math>N \in N_p</math> is a superset of a member <math>G \in N_p</math>, where <math>G</math> is an open set.</p>	08	CO2
		08	CO3
		08	CO2
Q.4)	Attempt any two of the following: (six marks each)	12 Marks	CO
	<p>a) Define locally connected topological space with an example. A topological space <math>X</math> is locally connected and continuous then prove that it is continuous and locally connected in <math>Y</math>.</p> <p>(b) Define path connected space with an example. Prove that path connected space is connected.</p> <p>(c) Define compactness of a topological space with an example and prove that A topological space <math>(X, T)</math> is compact iff any family of closed sets having the finite intersection property has a non-empty intersection.</p>	06	CO4
		06	CO4
		06	CO4

Q.5)	Attempt any three of the following: (four marks each)	12 Marks	CO
	<p>a) Define open map in topological spaces? Let <math>X</math> and <math>Y</math> be any two topological spaces and <math>f: X \rightarrow Y</math> be a open function then prove that the following statements are equivalent.</p> <p>(i) <math>f(A^\circ) \subseteq [f(A)]^\circ, \forall A \subseteq X</math>.</p> <p>(ii) The image of each member of a basis for <math>X</math> is open in <math>Y</math>.</p> <p>b) Let <math>[a,b]</math> be closed interval then verify that <math>[a,b]</math> is a closed set.</p> <p>c) Define Restriction of a function. Let <math>X</math> and <math>Y</math> be any two topological spaces. Then show that</p> <p>1) <math>f: X \rightarrow Y</math> be a continuous function</p> <p>2) <math>(f _A): A \rightarrow Y</math> is continuous function, where <math>A</math> be a subset of <math>X</math>.</p> <p>d) Prove that Continuous image of a path connected space is path connected.</p>	<p>04</p> <p>04</p> <p>04</p> <p>04</p>	<p>CO3</p> <p>CO1</p> <p>CO3</p> <p>CO3</p>
Q.6)	Attempt any four of the following: (seven marks each)	28 Marks	CO
	<p>(a) Define <math>T_0</math>-space with an example. Let <math>X</math> and <math>Y</math> be any two topological space then prove that <math>X \times Y</math> is <math>T_0</math> if and only if <math>X</math> and <math>Y</math> are <math>T_0</math>.</p> <p>(b) Define <math>T_1</math>-space with an example and prove that every subspace of <math>T_1</math> space is <math>T_1</math></p> <p>(c) Define <math>T_2</math>-space with an example and prove that Hausdorff topologies are invariant under closed bijections.</p> <p>(d) Let <math>X</math> and <math>Y</math> be any two topological space then prove that <math>X \times Y</math> is locally connected if and only if <math>X</math> and <math>Y</math> are locally connected.</p> <p>e) Define topological space with an example and Write down the properties of open sets and closed sets with an examples.</p>	<p>07</p> <p>07</p> <p>07</p> <p>07</p> <p>07</p>	<p>CO5</p> <p>CO5</p> <p>CO5</p> <p>CO5</p> <p>CO1</p>