Sanjay Ghodawat University, Kolhapur 2017-18

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FY M Sc.	**School of Science**	**Semester II**
MTS 506	**Classical Mechanics**	**Max Marks: 100**
May 2018	**End Semester Examination (ESE)**	**Time: 3 Hrs.**

26 May 2018
10:30 AM to 1:30 PM

Instructions for Students: 1) Use of non-programmable calculator is allowed
2) All questions are compulsory

			Marks	Cos
Q1		Choose the correct alternative for the following questions		
	a)	A body continues in its state of rest or uniform motion, unless no external force is applied to it is (1) Law of inertia (2) Law of force (3) Law of action & reaction (4) none of the above	2	CO1
	b)	If the Lagrangian $L(x,\dot{x})$ corresponding to Atwood's machine is given as: $L(x,\dot{x})=\frac{1}{2}(m_1+m_2)\dot{x}^2+m_1gx+m_2g(l-x)$, where m_1, m_2, l and g are constants. Then the equation of motion is (1) $\dot{x}=\frac{m_1-m_2}{m_1+m_2}g$ (2) $\dot{x}=\frac{m_1+m_2}{m_1-m_2}g$ (3) $\ddot{x}=\frac{m_1+m_2}{m_1-m_2}g$ (4) $\ddot{x}=\frac{m_1-m_2}{m_1+m_2}g$	2	CO2
	c)	Whenever the Lagrangian for a system does not contain a coordinate q_k explicitly, (1) q_k is cyclic coordinate . (2) p_k is cyclic coordinate . (3) p_k is constant of motion. (4) None of the above.	2	CO3
	d)	For an elliptical orbit as seen from the sun which of the following remains constant? (1) Speed (2) Kinetic energy (3) Energy (4)Angular momentum	2	CO4
	e)	For conservative system, Hamiton principle function S and Hamilton characteristic function W satisfy (1) $S=W$ (2) $S=W-Et$ (3) $S=W+Et$ (4) $W=S+Et$	2	CO5
	f)	If $[X,Y]$ is the Poisson Bracket, then which of the following is not true (1) $[X,Y+Z]=[X,Y]+[X,Z]$ (2) $[X,Y]=-[Y,X]$ (3) $[X,YZ]=[X,Y]Z+Y[X,Z]$ (4) $[X,X]=1$	2	CO5

Q2		Attempt any two of the following		
	a)	Obtain Lagrange's equation of motion from D' Alembert's principle for non-conservative system.	10	CO2
	b)	Derive Hamilton's principle for non-conservative system from D'Alembert's principle and hence deduce Lagrange's equation of motion for conservative system from Hamilton's principle.	10	CO3
	c)	Find the orbit described by the planet under the inverse square law of attractive force. Classify the orbit on the basis of total energy.	10	CO4
Q3		Attempt any two of the following		
	a)	If the total external force acting on the system is zero, then show that the total linear momentum of the system is conserved.	8	CO1
	b)	Define the Hamiltonian and obtain Hamilton's canonical equations of motion from Hamilton's principle.	8	CO3
	c)	Define Poisson bracket of two dynamical variables. For any three dynamical variables u, v, w ,show that $[u,[v,w]]+[v,[w,u]]+[w,[u,v]]=0$.	8	CO5
Q4		Attempt any two of the following		
	a)	Explain the motion of particle falling under the action of gravity near the surface of earth.	6	CO1
	b)	Find the extremals for the isoperimetric problem $I(y(x))=\int_0^{\pi}(y'^2-y^2)dx$ subject to the conditions that $\int_0^{\pi} y\,dx=1$ and $y(0)=0$, $y(\pi)=1$.	6	CO2
	c)	Prove that the Poisson's bracket is invariant under canonical transformation.	6	CO5
Q5		Attempt any three of the following		
	a)	A particle is constrained to move in a circle in a vertical (xy) plane. Apply the D' Alembert's principle and show that for equilibrium $\ddot{x}y-\ddot{y}x-gx=0$.	4	CO1
	b)	Find the extremal of the functional $\int_1^2\left(\frac{x^3}{y'^2}\right)dx$ subject to the conditions that $y(1)=0$, $y(2)=3$.	4	CO2

c)	Find the central force under the action of which a particle will follow $r = a(1+\cos\theta)$ orbit.	4	CO4
d)	Determine the canonical transformation defined by the generating function $F(q,Q,t) = \frac{1}{2}m\omega(t)q^2 \cot Q$. Also find new Hamiltonian K where $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$.	4	CO5

Q6 Attempt any four of the following

a)	If the external and internal forces are both conservative, then show that the total potential energy V of the system is given by $V = \sum_i V_i^{(e)} \frac{1}{2}\sum_{i,j} V_{ij}^{(\text{int})}$, where $V_i^{(e)}$ is the potential energy due to the external force $\bar{F}_i^{(e)}$ and $V_{ij}^{(\text{int})}$ is the potential energy due to the internal force.	7	CO1
b)	Show that the non-conservation of total energy is directly associated with the existence of non-conservative forces even if the transformation equation does not contain time t.	7	CO2
c)	If the cyclic generalized coordinate q_j is such that dq_j represents the translation of the system, then prove that the total linear momentum is conserved.	7	CO3
d)	A particle of mass m is moving under the inverse square law of attractive force. Set up the Lagrangian and equation of motion. Obtain the first integral of motion and also show that it is energy constant.	7	CO4
e)	Prove the relation $\frac{dA}{dt} = \frac{\partial A}{\partial t} + [A,H]$. Represent the equation of motion using Poisson bracket. Show if A is independent of time & [A, H]=0, then A is constant of motion.	7	CO5

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