Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra.
Act No XL, 2017

2017-18

FY M Sc	**School of Sciences | Department of Mathematics**	**Semester-II**
MTS 502	**Algebra-I**	**Max Marks:100**
22/5/18 10:30 Am To 1:30pm	**End Semester Examination(ESE)**	**Time 3Hr**

Instructions for Student: 1) All Questions are compulsory.

2) Use non programmable calculator is allowed

			Marks	CO
Q.1		Choose correct alternative for the following questions (12)		
	a)	Statement I :Every normal series is a subnormal series. Statement II:Every subnormal series is a normal series . Then, which of the following statement is correct ? A)Only statement I is true. B)Only statement II is true C)Both statement are true D)Both statement are false	2	CO1
	b)	Let G be a finite group. Statement I :If G is a p-group then $\|G\|$ is a power of prime p. Statement II:If $\|G\|$ is a power of prime p then G is a p-group. Then, which of the following statement is correct ? A)Only statement I is true. B)Only statement II is true C)Both statement are true D)Both statement are false	2	CO2
	c)	Let I_1 be the ideal generated by x^4+3x^2+2 and I_2 be the ideal generated by x^3+1 in $Q[x]$.If $F_1=\frac{Q[x]}{I_1}$ and $F_2=\frac{Q[x]}{I_2}$, Then, which of the following statement is correct ? A) F_1 and F_2 are fields. B) F_1 is a field, but F_2 is not a field. C) F_1 is not a field while F_2 is a field. D)Neither F_1 nor F_2 is a field.	2	CO3
	d)	Statement I: Hilbert basis theorem holds in Artinian rings. Statement II: Hilbert basis theorem holds in Noetherian rings. Then, which of the following statement is correct?	2	CO4

A) I and II both are correct B) I and II both are incorrect

C) Only I is Correct D) Only II is Correct

e) Statement I: Maximal condition holds in Artinian rings. 2 CO4

Statement II: Maximal condition holds in Noetherian rings.

Statement III: Minimal condition holds in Artinian rings.

Statement IV: Minimal condition holds in Noetherian rings.

Then, which of the following statement is correct?

A) I and II both are correct B) II and III both are correct

C) I and IV both are correct D) III and IV both are correct

f) Statement I: Union of two submodule is again a submodule. 2 CO5

Statement II: Intersection of two submodules is again a submodule

Then, which of the following statement is correct?

A) I and II both are correct B) I and II both are incorrect

C) Only I is Correct D) Only II is Correct

Q.2 Attempt any two of the following. (20 marks)

a) Let H and K be subgroups of group G. H^* and K^* be normal subgroups of H and K respectively. Then show that, 10 CO1

i) $H^*(H\cap K^*)$ is a normal subgroup of $H^*(H\cap K)$.

ii) $K^*(H^*\cap K)$ is a normal subgroup of $K^*(H\cap K)$.

iii) $\dfrac{H^*(H\cap K)}{H^*(H\cap K^*)} \cong \dfrac{K^*(H\cap K)}{K^*(H^*\cap K)} \cong \dfrac{H\cap K}{(H^*\cap K).(H\cap K^*)}$

b) Let G be a finite group with $|G| = p^n.m$ where p is a prime number and $p \nmid m$. Then, show that 10 CO2

i) G contains a subgroup of order p^i for each i, $1 \le i \le n$

ii) Every subgroup of order p^i is a normal subgroup of a subgroup of order p^{i+1} for $1 \le i \le n-1$.

c) Let F be a field . Let $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n$ and $g(x)=b_0+b_1x+b_2x^2+\cdots+b_mx^m$ be two polynomials in F[x] with $a_n \neq 0$ and $b_m \neq 0$ with m >0. Then, show that there are two polynomials q(x) and r(x) in F[x] such that f(x)=q(x) g(x)+r(x) with deg r(x)<deg g(x). These polynomial q(x) and r(x) are unique 10 CO3

Q.3 Attempt any **Two** (16 marks)

a)	R[x] is a ring of polynomial over R. Show that R[x] is commutative iff R is commutative.	8	CO3
b)	Show that following statements are equivalent in R. i)R satisfies a.c.c for ideals(R is Noetherian ring) ii)The maximum condition holds in R iii)Every ideal in R is finitely generated	8	CO4
c)	Show that homomorphic image of an R-module M is isomorphic with its suitable quotient module.	8	CO5

Q.4 Attempt any **Two** of the following. (12 marks)

a)	Show that two subnormal series of a group G have isomorphic refinements.	6	CO1
b)	Let G be a finite group. Then show that G is p-group iff $\lvert G\rvert$ is a power of prime p.	6	CO2
c)	Let M be any R-module. For any two submodules N_1 and N_2 of M, Show that N_1+N_2 is a submodule of M, containing N_1 and N_2 both.	6	CO5

Q.5 Attempt any **Three** of the following. (12 marks)

a)	Show that any two composition series of a group G are isomorphic.	4	CO1
b)	Let G be a finite group with $\lvert G\rvert=pq$ where p and q are distinct primes and $p<q$. Then show that G contains a normal subgroup of order q	4	CO2
c)	Show that any Artinian domain R is a field.	4	CO4
d)	Let M be any R-module. Then, show that i)$0.m=0$ for all $m \in M$ ii)$r.0=0$ for all $r \in R$ iii)$(-r).m=(-rm)=r.(-m)$ for all $r \in R$ 7	4	CO5

Q.6 Attempt any **Four** of the following. (28 marks)

a)	Let G and G' be group and let $\phi: G \to G'$, be an onto homomorphism. Then, show that $G' \cong \frac{G}{Ker\phi}$.	7	CO1
b)	State and prove Burnside theorem.	7	CO2
c)	Show that the following polnomial is irreducible over Q i) $x^3+x^2-2x-1 \in Z[x]$, ii) $x^4+x^3+x^2+x+1 \in Z[x]$	7	CO3
d)	If R is Noetherian ring, then show that any homomorphic image of R is also Noetherian	7	CO4

e) Let A and B be R-submodules of an R-module M. 7 CO5

Then show that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$