



# Sanjay Ghodawat University, Kolhapur

2017-18

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

FY M Sc.

School of Science

Semester II

MTS ~~509~~ 510

Partial Differential Equations

Max Marks: 100

May/June 2018

End Semester Examination (ESE)

Time: 3 Hrs.

2 June 2018

**Instructions for Students:** 1) All questions are compulsory  
10:30 AM to 1:30 PM.

Q1	Choose correct alternative for the following questions	Marks	COs
a)	Solution of $p^2 + q^2 = x + y$ is.....	2	CO1
	a. $z = (a+x)^{3/2} + \frac{3}{2}(y-a)^{3/2} + b$		
	b. $z = \frac{2}{3}(a+x)^{3/2} + \frac{2}{3}(y-a)^{3/2} + b$		
	c. $z = \frac{3}{2}(a+x)^{3/2} + \frac{3}{2}(y-a)^{3/2} + b$		
	d. None of these		
b)	A. The solution of ordinary differential equation of order n have n arbitrary constants B. The solution of partial differential equation of order n has n arbitrary functions a. A is true, B is false      b. A is false, B is true c. Both A and B are true      d. Both A and B are false	2	CO1
c)	The relation $z = (x+a)(y+b)$ represents the partial differential equation .....	2	CO2
	a. $z = \frac{p}{q}$ b. $z = pq$		
	c. $z = p - q$ d. none of these		
d)	A solution of $u_{xx} = 0$ is of the form.....	2	CO3
	a) $u = \int f(y)dx + \phi(y)$ b) $u = \int (f'(y)dy + f(y)dx)$		
	c) $u = \int f(y)dx$ d) $u = \int (f(x)dx + f(y)dy)$		
e)	The heat equation is.....	2	CO4
	a) $\frac{\partial u}{\partial x} = c^2 \nabla^2 x^2$ b) $\frac{\partial u}{\partial y} = \frac{\nabla^2 t^2}{c^2}$		
	c) $\frac{\partial u}{\partial t} = c^2 \nabla^2 u$ d) $\frac{\partial u}{\partial t} = c^2 \nabla^2 y^2$		
f)	The differential equation $f_{xx} + 2f_{xy} + 4f_{yy} = 0$ is classified as	2	CO3

- a) Hyperbolic  
c) Elliptic

- b) Parabolic  
d) None of these

- Q2** Attempt any two of the following
- a) By a suitable change of independent variables, show that a second order p.d.e  $Rr + Ss + Tt + g(x, y, x, u_x, u_y) = 0$  can be reduced to one of the canonical form on the basis of  $S^2 - 4RT = 0$  10 CO3
- b) By separable variable method, find the solution equation 10 CO4  

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, 0 < x < l, t > 0$$
 satisfying the conditions  
 $u(0, t) = 0, u(l, t) = 0, t > 0$  and  $u(x, 0) = f(x), 0 < x < l$
- c) Solve  $\nabla^2 u = 0, 0 \leq x \leq a, 0 \leq y \leq b$ , subject to the boundary condition 10 CO5  
 $u_x(0, y) = u_x(a, y) = 0, u_y(x, 0) = 0, u_y(x, b) = f(x)$
- Q3** Attempt any two of the following
- a) Find the complete integrals by charpits method 8 CO2  
 $(p^2 + q^2)y - qz = 0$
- b) Find the condition that a one parameter family of surfaces form a family of equipotential surfaces 8 CO4
- c) Solve  $\nabla^2 u = 0, r < a$  subject to the boundary condition 8 CO5  

$$\frac{\partial u}{\partial n} = \frac{\partial u}{\partial n} = f(\theta) \text{ on } r = a \text{ where } \int_0^{2\pi} f(\theta) d\theta = 0$$
- Q4** Attempt any two of the following
- a) Find the direction cosines of the tangent of the curve 6 CO1  
 $x = x(s), y = y(s), z = z(s)$  where  $s$  is arc length of the curve measured from the fixed point  $P_0$  on the curve to any point  $P$  on the curve are  

$$\left( \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds} \right)$$
- b) Solve the equations by Jacobi's method  $z^2 + zu_z - u_x^2 - u_y^2 = 0$  6 CO2
- c) Write short note on classification of second order p.d.e and its canonical forms. 6 CO3
- Q5** Attempt any three of the following
- a) Find the direction cosines of the normal to the surface  $S$  of the form 4 CO1  
 $z = f(x, y)$

- b) Obtain the partial differential equation of first order by elimination of arbitrary constants from the relation  $(x-a)^2 + (y-b)^2 + z^2 = 1$  4 CO1
- c) Write short note on types of some standard types of first order differential equations 4 CO2
- d) Prove that the solution of the Dirichlet problem, if it exists, is unique 4 CO5

**Q6**

Attempt any four of the following

- a) Show that following Pfaffian differential equations are integrable and hence find its corresponding integral. 7 CO1
- $$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$
- b) Describe the Charpit's method of solving a first order partial differential equations  $f(x,y,z,p,q)=0$  7 CO2
- c) Reduce the equations to Canonical form  $x^2u_{xx} - y^2u_{yy} = 0$  7 CO3
- d) Show that the following surfaces form a family of equipotential surfaces and find general form of the corresponding potential function 7 CO4
- $$(x^2 + y^2)^2 - 2a^2(x^2 + y^2) + a^4 = c$$
- e) Stability Theorem : Show that the solution of the Dirichlet problem is stable 7 CO5

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