SGU

Sanjay Ghodawat University, Kolhapur

2017-18

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FY M. Sc.	**School of Science**	
MTS 504	**Complex Analysis**	**Semester II**
May 2018	**End Semester Examination (ESE)**	**Max Marks: 100**
24th May 18		**Time: 3 Hrs.**

Instructions for Students: 1) Use of non-programmable calculator is allowed
10:30 AM TO 1:30 PM. 2) All questions are compulsory

Q1

Attempt the following: (two marks each) — **Marks** **COs**

a) The power series $\sum_{n=0}^{\infty} 3^{-n} z^{2n}$ converges if — 02 — CO1

A) $|z| \le 3$
B) $|z| < 3$
C) $|z-1| < \sqrt{3}$
D) $|z-1| \le \sqrt{3}$

b) Let C be the circle $|z| = \frac{3}{2}$ in the complex plane that is oriented in the counter clockwise direction. The value of a for which $\int_C \left(\frac{z+1}{z^2 - 3z + 1} + \frac{a}{z-1} \right) dz = 0$ — 02 — CO2

A) -1
B) 1
C)
D) -2

c) A continuous curve which does not have a point of self intersection is called — 02 — CO2

A) Simple curve
B) Rectifiable curve
C) Integral curve
D) Close Curve

d) The value of $\int_C \frac{3z^2 + 7z + 1}{z+1} dz$,where C is the circle $|z| = \frac{1}{2}$ is — 02 — CO3

A) $2\pi i$
B) πi
C) $\frac{\pi i}{2}$
D) 0

e) The poles of the function $f(z) = \frac{\sin z}{\cos z}$ are at — 02 — CO4

A) $\frac{(2n+1)\pi}{2}, n$ is integer
B) $\pi, -\pi$
C) $n\pi, n$ is integer
D) $\frac{n\pi}{2}, n$ is integer

f) The number of zeros of $z^3 - 6z + 8 = 0$ inside the circle $|z| = 1/2$ is — 02 — CO5

A) 0
B) 1
C) 2
D) 3

Q2 **Attempt any two of the following: (ten marks each)**

a) If $f(Z)=\sum_{n=0}^{\infty} a_n (z-a)^n$ have radius of convergence $R>0$. Then prove that — 10 — CO1

(i) for each $k \geq 1$ the series $\sum_{n=k}^{\infty} n(n-1)...(n-k+1)a_n(z-a)^{n-k}$ has radius of convergence R

(ii)The function f is infinitely differentiable on $B(a;\ R)$ and further $f^{(k)}(z)$ given by $f^{(k)}(z)=\sum_{n=k}^{\infty} n(n-1)...(n-k+1)a_n(z-a)^{n-k}$ has radius of convergence R

(iii) for $n \geq 0$; $a_n = \frac{1}{n!} f^{(n)}(a)$

b) Let $\phi:[a,b]\times[c,d]\to C$ be a continuous function and $g:[c,d]\to C$ by $g(t)=\int_a^b \phi(s,t)ds$. Then prove that (i) g is continuous. (ii) if $\frac{\partial \phi}{\partial t}$ exists and is a continuous function on $[a,b]\times[c,d]$ then g is continuously differentiable and $g'(t)=\int_a^b \frac{\partial \phi}{\partial t} ds$ — 10 — CO2

c) State and prove Cauchy theorem — 10 — CO3

Q3 **Attempt any two of the following: (eight marks each)**

a) Evaluate — 08 — CO3

(i) $\int_{|z|=2} \frac{z-3\cos z}{\left(z-\pi/2\right)^2} dz$ (ii) $\int_{|z|=3} \frac{e^z}{(z-2)} dz$

b) Let $z=a$ be an isolated singularity of a function *f*, then prove that $z=a$ is a removable singularity of *f* iff $\lim_{z\to a}(z-a)f(z)=0$ — 08 — CO4

c) State and prove Roche's theorem. — 08 — CO5

Q4 **Attempt any two of the following: (six marks each)**

a) Prove that the image of real axis under any möbious transformation is either a circle or straight line. — 06 — CO1

b) State and prove open mapping theorem. — 06 — CO4

c) Show that there are three roots of the equation $z^3-6z+8=0$ lie between the circle $|z|=1$ and $|z|=3$ — 06 — CO5

Q5 **Attempt any three of the following: (four marks each)**

		Marks	CO
a)	Find the radius convergence of the series $\sum_{n=0}^{\infty}(5+12i)^n z^n$	04	CO1
b)	Let *f(z)* be analytic in *B(a; R)* and suppose $\mid f(z)\mid\leq M,\ z\in B(a;R)$ then prove that $\mid f^n(a)\mid\leq n!\frac{M}{R^n}$	04	CO2
c)	Write statement of following theorem (i) Morera's theorem (ii)Liouville's theorem	04	CO3
d)	Evaluate $\int_{\mid z\mid=3/2}\frac{\cos(\pi z)}{\sin(\pi z)}dz$	04	CO4

Q6 **Attempt any four of the following (seven marks each)**

		Marks	CO
a)	State and prove the Necessary condition for *f (z)* to be analytic.	07	CO1
b)	If $\mid z\mid<1$, prove that $\int_0^{2\pi}\frac{e^{is}}{e^{is}-z}ds=2\pi$	07	CO2
c)	Let $\gamma:[0,1]\to C$ be a closed rectifiable curve and $a\notin\{\gamma\}$ then prove that $\frac{1}{2\pi i}\int_\gamma\frac{dz}{(z-a)}$ is an integer.	07	CO3
d)	Evaluate $\int_\gamma\frac{e^z}{(z-1)^4}dz;\ \gamma:\mid z-1\mid=1$ by using Cauchy Residue theorem	07	CO4
e)	Let *f* is analytic on *D* with $\mid f(z)\mid\leq 1$ and let $f(a)=\alpha$ for $a\in D=\{z:\mid z\mid<1\}$ then prove that $\mid f'(a)\mid\leq\frac{1-\mid\alpha\mid^2}{1-\mid a\mid^2}$. Further if $\mid f'(a)\mid=\frac{1-\mid\alpha\mid^2}{1-\mid a\mid^2}$ then there is a constant c with $\mid c\mid=1$ and $f(z)=\phi_{-\alpha}(c\phi_a(z))$ for $z\in D$	07	CO5

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