Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

2018-19
EXM/P/09/01

Year and Program:2018-19 B.Sc.II	**School of Science**	**Department of Mathematics**
Course Code: MTS 201	**Course Title: Mathematics III**	**Semester – III**
Day and Date: Tuesday 4 Dec18	**End Semester Examination (ESE)**	**Time:** 2:30pm to 5:30pm
		Max Marks: 100
PRN/Exam seat No:		**Answer booklet No:**
Student's signature:		**Invigilator signature:**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question has four choices out of which only one is the correct
4) Tick mark (√) the correct alternative which should be answered in question paper itself and submit to the Jr.Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicates full marks
7) Use **Blue ball pen** only.

Q.1	**Choose the correct Alternative for following questions.**	Marks	Bloom's Level	CO
i)	If A,B,C are sets, then I) $A\setminus(B\cup C)=(A\setminus B)\cup(A\setminus C)$ II) $A\setminus(B\cap C)=(A\setminus B)\cap(A\setminus C)$ a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L2	CO1
ii)	Suppose that S and T are sets and that $T\subseteq S$, then I) If S is a finite set, then T is a finite set. II) If T is infinite set, then S is an infinite set. a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L2	CO1

iii)	If r is rational number and x is irrational number, then	01	L2	CO1
	a) $r+x$ & rx are rational b) $r+x$ & rx are irrational			
	c) rx is only rational d) $r+x$ is only rational			
iv)	Let A and B are two irrational numbers then	01	L2	CO1
	a) A+B is always irrational number			
	b) A+B is always rational number			
	c) AB is always rational			
	d) AB may be rational			
v)	Let $A=\left\{-2,\frac{-3}{2},\frac{-4}{3},\frac{-5}{4},.....,\frac{-n+1}{n},.......\right\}$ then inf A=......	01	L2	CO2
	a) 0 b) 2 c) -2 d) 1/2			
vi)	I) If x and y are complex then $\left\|\|x\|-\|y\|\right\|=\|x-y\|$	01	L2	CO2
	II) If $x\in R^k$ and $y\in R^k$ then $\|x-y\|^2+\|x+y\|^2=2\|x\|^2+2\|y\|^2$			
	a) Both I and II true b) only I true			
	c) Both I and II false d) only II true			
vii)	The set A of all real numbers x such that $2x+3\le 6$ then	01	L2	CO2
	a) $A=\left\{x\in R:x\le \frac{3}{2}\right\}$ b) $A=\left\{x\in R:x<\frac{3}{2}\right\}$			
	c) $A=\left\{x\in R:x>\frac{3}{2}\right\}$ d) $A=\left\{x\in R:x\ge \frac{3}{2}\right\}$			
viii)	If $a\in\mathbb{R}$ is such that $0\le a<\in$ for every $\in>0$, then	01	L2	CO2
	a) $a<0$ b) $a=0$ c) $a=\in$ d) None of these			

ix)	$Let\, a_1 = -2, a_{n+1} = \frac{na_n}{n+1}$ $then \lim_{n\to\infty} a_n = \ldots.$	01	L2	CO3
	a) 1 b) -1 c) 0 d) does not exist			
x)	I) Every bounded sequence in R contains convergent subsequence. II) A sequence $\{P_n\}$ Converges to p, if every subsequence of $\{P_n\}$ converges to p. a) Both I and II true b) only I true c) Both I and II false d) only II true	01	L2	CO3
xi)	If $c>0$ then $\lim_{n\to\infty}\left(c^{\frac{1}{n}}\right) = ------$.	01	L2	CO3
	a) 1 b) 0 c) -1 d) Not defined			
xii)	$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \ldots\ldots\ldots$	01	L2	CO3
	a) 1 b) 0 c) -1 d) e			
xiii)	If the series $\sum_{n=1}^{\infty} x_n$ converges then $\lim(x_n) = \ldots..$	01	L2	CO4
	a) 0 b) 1 c) -1 d) 1/2			
xiv)	I) If $\sum a_n$ converges then $\lim_{n\to\infty} a_n = 0$ II) If $\lim_{n\to\infty} a_n = 0$ then $\sum a_n$ converges. a) Both I and II true b) only I true c) Both I and II false d) only II true	01	L2	CO4

xv)	I) $n<2^n\ \forall n\in N$, II) $2^n<n!\ \forall n\in N$	01	L2	CO4

a) Both I and II true b) only I true

c) Both I and II false d) only II true

xvi)	If $S_n=1+\left[\frac{(-1)^n}{n}\right]$ then sequence $\{S_n\}$ is	01	L2	CO4

a) converges to 1,bounded & has infinite range

b) converges to 0, bounded & has infinite range

c) converges to 1, bounded and has finite range

d) converges to 0, bounded and has finite range

xvii	$\lim\limits_{n\to\infty}\frac{x^2+nx}{n}=$........$\forall x\in R$	01	L2	CO5

a) 1 b) 0 c) -1 d) x

xviii	I) Every Cauchy sequence of real number is bounded.	01	L2	CO5

II) Every Bounded sequence of real number is Cauchy.

a) Both I and II true b) only I true

c) Both I and II false d) only II true

xix)	$\lim\limits_{n\to\infty}\frac{3x^2+7}{n}=$........., $\forall x\in\mathbb{R}$	01	L2	CO5

a) 1 b)0 c)-1 d) Does not exist

xx)	$\lim\limits_{n\to\infty}\frac{nx}{1+n^2x^2}=$.........; *for all* $x\in\mathbb{R}$	01	L2	CO5

a) 1 b) 0 c) -1 d) e

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		Max Marks: 100

Instructions:
1) All questions are compulsory.
2) Assume suitable data wherever necessary.
3) Figures to the right indicate full marks.

			Marks	Bloom's Level	CO
Q.2		**Solve any Two.**			
	a)	Prove that Every subset of countable set is countable.	5	L3	CO1
	b)	Prove that any open interval (a,b) is equivalent to any other open interval (c,d).	05	L3	CO1
	c)	Show that for each $n \in N$, the sum of the squares of the first n natural numbers is given by $\frac{1}{6}n(n+1)(2n+1)$	05	L3	CO1
Q3		**Solve any Two.**			
	a)	If x and y are any real numbers with property $x<y$,then prove that there exist a rational number $r \in \mathbb{Q}$ such that $x<r<y$	05	L4	CO2
	b)	If $x>-1$ then prove that $(1+x)^n \geq 1+nx$	05	L3	CO2
	c)	Determine the set A of $x \in \mathbb{R}$ such that $\|2x+3\|<7$	05	L3	CO2
Q4		**Solve any Two.**			
	a)	Let $X=(x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n>0$. Then show that the sequence $\sqrt{x_n}$ of positive square roots converges and $lim\left(\sqrt{x_n}\right)=\sqrt{x}$.	05	L4	CO3

b)	Let $X=(x_n)$ be a bounded sequence of real numbers and let $x\in\mathbb{R}$ have that property that every convergent subsequence of X converges to x. Then prove that the sequence X converges to x.	05	L4	CO3
c)	Show that, a monotone sequence of real numbers is convergent if and only if it is bounded. Further if $X=(x_n)$ is bounded increasing sequence, then $\lim(x_n)=\sup\{x_n : n\in\mathbb{N}\}$	05	L4	CO3

Q5

a)	**Solve any Three.**			
i)	i)If a series $\sum x_n$ is convergent, then show that any series obtained from it by grouping the terms is also convergent and to the same value.	06	L3	CO4
ii)	Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ convergent.	06	L3	CO4
iii)	Establish the convergence or the divergence of the series whose nth term is $\frac{n}{(n+1)(n+2)}$	06	L3	CO4
iv)	Test the given series for convergence and for absolute convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$	06	L3	CO4
b)	Show that $\sum_{n=1}^{\infty}\frac{1}{n^2}$ Series is convergent	07	L3	CO4
	OR			
b)	Test the given series for convergence and for absolute convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n}{n+2}$	07	L3	CO4

Q.6

a)	**Solve any Three.**			
i)	Let (f_n) be a sequence of functions in $\Re[a,b]$ and suppose that (f_n) converges uniformly on $[a,b]$ to f. Then prove that $f \in \Re[a,b]$ and $\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$.	06	L4	CO5
ii)	Show that $\lim_{n\to\infty}\left(\frac{x^2+nx}{n}\right) = x$ for $x \in \mathbb{R}$.	06	L3	CO5
iii)	Show that $\lim_{n\to\infty}\left(\frac{1}{n}\sin(nx+n)\right) = 0$ for $x \in \mathbb{R}$.	06	L3	CO5
iv)	Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that converges (f_n) uniformly on A to a function $f : A \to \mathbb{R}$. Then show that f is continuous on A.	06	L4	CO5
b)	A sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.	07	L4	CO5
	OR			
b)	Let $J \subseteq \mathbb{R}$ be a bounded interval and let (f_n) be a sequence of functions on J to $\mathbb{R}$. Suppose that there exists $x_0 \in J$ such that $(f_n(x_0))$ converges and that the sequence (f_n') of derivatives exists on J and converges uniformly on J to a function g. Then show that the sequence (f_n) converges uniformly on J to a function f that has a derivative at every point of J and $f' = g$.	07	L4	CO5

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