Sanjay Ghodawat University, Kolhapur

2017-18

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

FY B Tech | School of Technology | Semester I

FYT 101 | Matrices & Multivariable Calculus | Max Marks: 100

Nov 2017 | Re - End Semester Examination (ESE) | Time: 3 Hrs

26 Dec

Instructions for Students: 1) Use of non-programmable calculator is allowed
2) All questions are compulsory

Q1 Solve the following — Marks — COs

a) Reduce the following matrix to normal form and hence find the rank — **05** — CO-1

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

b) Solve: $4x+3y-z=0, 3x+4y+z=0, x-y-2z=0, 5x+y-4z=0$ — **05** — CO-1

c) For what value of λ and μ the system of equations $x+y+z=6$, $x+2y+5z=10$, $2x+3y+\lambda z=\mu$ has i) a unique solution, ii) infinitely many solutions and iii) no solution. Also find solution when $\lambda=2$ and $\mu=8$ — **06** — CO-1

OR

c) Determine λ such that the system of equations $2x+y+2z=0$, $x+y+3z=0$, $4x+3y+\lambda z=0$ has i) trivial solution ii) Non-trivial solution. Also find the non-trivial solution — **06** — CO-1

Q2 Solve the following

a) Show that the vectors $(1\ 2\ 4), (2\ -1\ 3), (0\ 1\ 2), (-3\ 7\ 2)$ are linearly dependent. Express one of the vector as linear combination of the others. — **05** — CO-2

b) Using Cayley-Hamilton theorem find A^4, where $A=\begin{bmatrix} 1 & 1 & -1 \\ -1 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ — **05** — CO-2

c) Find Eigen values of the matrix $A=\begin{bmatrix} 1 & 3 & -3 \\ -3 & 7 & -3 \\ -6 & 6 & -2 \end{bmatrix}$. Also find Eigen vector corresponding to greatest Eigen value — **06** — CO-2

OR

c) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}3 & 2 & 4\\ 4 & 3 & 2\\ 2 & 4 & 3\end{bmatrix}$ and hence find A^{-1} — **06** — CO-2

Q3 Attempt **Any Three** from the following

a) If $u=\log(x^2+y^2)+\tan^{-1}\left(\frac{y}{x}\right)$, find the value of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$ — **06** — CO-3

b) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$ and ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\cos 3u\sin u$ — **06** — CO-3

c) If $x=e^v\sec u$ and $y=e^v\tan u$ then show that $\frac{\partial(u,v)}{\partial(x,y)}\cdot\frac{\partial(x,y)}{\partial(u,v)}=1$ — **06** — CO-3

d) If $z=f(x,y)$ and $x=e^u\cos v, y=e^u\sin v$, prove that i). $x\frac{\partial z}{\partial v}+y\frac{\partial z}{\partial u}=e^{2u}\frac{\partial z}{\partial y}$ and ii) $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2\right]$ — **06** — CO-3

Q4 **Solve the following**

a) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surrounded by hemispherical ends. If the radius is increased by 0.01 m and length by 0.05 m. Find the percentage change in the volume of balloon, where volume of balloon is $V=\pi r^2 h+\frac{4}{3}\pi r^3$ — **05** — CO-4

b) Find extreme values of $x^2+y^2+xy+x-4y+5$ — **05** — CO-4

c) Prove that $\int_0^\infty \frac{e^{-ax}\sin x}{x}\,dx=\cot^{-1}a$ and hence find the value of $\int_0^\infty \frac{\sin x}{x}\,dx$ — **06** — CO-4

OR

c) The temperature T at any point (x,y,z) of space is given by $T=400xyz^2$, find the highest temperature at the surface of sphere $x^2+y^2+z^2=1$ — **06** — CO-4

Q5 Attempt **Any Three** from the following

a) Evaluate $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r\sin\theta\, dr\, d\theta$ **06** CO-5

b) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\, dx\, dy\, dz$ **06** CO-5

c) Evaluate $\iint xy(x+y)dx\, dy$ over the area between $y=x^2$ and $y=x$ **06** CO-5

d) Change the order of integration and evaluate $\int_0^1\int_{4y}^{4} e^{x^2} dx\, dy$ **06** CO-5

Q6 **Solve the following**

a) Find the area between the circle $x^2+y^2=9$ and the line $x+y=3$ in the first quadrant. **05** CO-6

b) A lamina is bounded by the curves $y=x^2-3x$ and $y=2x$. If the density at any point is given by λxy, find the mass of lamina using double integration. **05** CO-6

c) Prove that the moment of inertia about an axis through the cemtre perpendicular to the plane of a circular ring whose outer and inner radii are a and b respectively is $\frac{1}{2}M(a^2-b^2)$, where M is mass of the ring. **06** CO-6

OR

c) Find the volume of the solid generated revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about y axis. **06** CO-6

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