Sanjay Ghodawat University, Kolhapur

2017-18

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

FY B Tech | School of Technology | Semester I

FYT 101 | Matrices and Multivariable Calculus | Max Marks: 100

27 Nov 2017 | End Semester Examination (ESE) | Time: 3 Hrs

Instructions for Students: 1) Use of non-programmable calculator is allowed
2) All questions are compulsory

Q1 Solve the following

		Marks	COs
a)	Reduce the following matrix to the normal form and find its rank $\begin{bmatrix} 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \\ 10 & 11 & 12 & 13 \\ 18 & 19 & 20 & 21 \end{bmatrix}$	05	CO1
b)	Test for consistency and if possible solve them $x+y+z=-3;\ 3x+y-2z=-2;\ 2x+4y+7z=7$	05	CO1
c)	Solve $x+y+2z=0;\ x+2y+3z=0;\ x+3y+4z=0;\ 3x+4y+7z=0$	06	CO1
	OR		
c)	Find the value of k such that the system of equations $2x+3y-2z=0;\ 3x-y+3z=0;\ 7x+ky-z=0$ has nontrivial solutions. Find the solution.	06	CO1

Q2 Solve the following

a)	Examine for linear dependence or independence of vectors $(1, 2, 3), (3, -2, 1), (1, -6, -5)$. If dependent find the relation between them.	05	CO2
b)	Find the Eigen values of $A=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and hence find Eigen vector corresponding to greatest Eigen value.	05	CO2
c)	Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1}, where $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	06	CO2

OR

c) Use Cayley-Hamilton theorem to find A^{-1} and A^4, where **06** CO2

$$A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Q3 Attempt any Three from the following

a) If $u=e^{xyz}$ then find $\dfrac{\partial^3 u}{\partial x\partial y\partial z}$ **06** CO3

b) If $a^2x^2+b^2y^2+c^2z^2=1$ and $x+y+z=k$; prove that **06** CO3

$$\frac{dx}{b^2y-c^2z}=\frac{dy}{c^2z-a^2x}=\frac{dz}{a^2x-b^2y}$$

c) If $u=\log\left(\dfrac{x^3+y^3}{x^2+y^2}\right)$; find (i) $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$ and **06** CO3

ii) $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y\dfrac{\partial^2 u}{\partial y^2}$

d) If $x=r\cos\theta,\ y=r\sin\theta$; prove that $JJ'=1$ **06** CO3

Q4 Solve the following

a) The power dissipated in a resistor is given by $P=\dfrac{E^2}{R}$. Using calculus, find the approximate percentage change in P, where E is increased by 3% and R is decreased by 2%. **05** CO4

b) Find extreme values of $x^3+3xy^2-3x^2-3y^2+4$ **05** CO4

c) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $xyz=a^3$ **06** CO4

OR

c) Prove that $\displaystyle\int_0^1\frac{x^\alpha-1}{\log x}dx=\log(1+\alpha);\ \alpha\geq 0$ **06** CO4

Q5 Attempt any three from the following

a) Evaluate $\displaystyle\int_0^2\int_0^{x^2} x(x^2+y^2)\,dydx$ **06** CO5

b) Evaluate $\displaystyle\int_0^a\int_0^x\int_0^{\sqrt{x+y}} z\,dzdydx$ **06** CO5

c) Change the order of integration and evaluate $\int_0^5 \int_{2-x}^{2+x} dydx$ **06** CO5

d) Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2+y^2=a^2$ **06** CO5

Q6 Solve the following

a) Find the area of the cardioid $r=a(1+\sin\theta)$ **05** CO6

b) Find the volume of solid formed by the area of revaluation about Y-axis, of the part of the parabola $y^2=4ax$ cut off by the latus rectum. **05** CO6

c) A lamina is bounded by the curves $y=x^2-3x$ and y=2x. If the density at any point given by λxy, find the mass of the lamina. **06** CO6

OR

c) Find the moment of inertia about X-axis of the area enclosed by the line $x=0, y=0$ and $\frac{x}{a}+\frac{y}{b}=1$. **06** CO6

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