



Sanjay Ghodawat University, Kolhapur

2017-18

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

FY B. Tech

School of Technology

Semester II

FYT 114

Complex Numbers and Calculus

Max Marks: 100

21 May 2018

End Semester Examination (ESE)

Time: 3 Hrs.

10:30AM - 01:30PM

- Instructions for Students: 1) Use of non-programmable calculator is allowed
2) All questions are compulsory

Q1	Solve the following	Marks	COs
a)	Simplify, $\frac{[\cos 5\theta - i \sin 5\theta]^2 [\cos 7\theta + i \sin 7\theta]^{-3}}{[\cos 4\theta - i \sin 4\theta]^9 [\cos \theta + i \sin \theta]^5}$.	05	CO1
b)	Show that, $\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$.	05	CO1
c)	Prove that n^{th} roots of unity are in Geometric progression and also show that the sum of the n^{th} roots of unity is zero.	06	CO1

OR

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| c) | Solve $x^5 = 1 + i$ and find the continued product of the roots. | 06 | CO1 |
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Q2	Attempt Any Three from the Following	Marks	COs
a)	If $\sin(\theta + i\phi) = r(\cos\alpha + i\sin\alpha)$, prove that, $r^2 = \frac{1}{2}[\cosh 2\phi - \cos 2\theta]$ and $\tan\alpha = \tanh\phi$.	06	CO2
b)	Show that $\tan^{-1}i \left[\frac{x-a}{x+a} \right] = \frac{i}{2} \log \left(\frac{x}{a} \right)$.	06	CO2
c)	Prove that $\cosh^{-1}z = \log(z + \sqrt{z^2 - 1})$.	06	CO2
d)	Prove that $i \log \left(\frac{x-i}{x+i} \right) = \pi - 2\tan^{-1}x$.	06	CO2

Q3	Solve the following	Marks	COs
a)	Using Comparison test, discuss the convergence of, $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$	05	CO3
b)	Using D'Alembert's ratio test, discuss the convergence of $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots, x > 0.$	05	CO3
c)	Using Cauchy's n^{th} root test, discuss the convergence of $\sum_1^{\infty} \left[\frac{n+1}{n+2} \right]^n \cdot x^n, x > 0.$	06	CO3

OR

c)	Examine the convergence of $\sum_1^{\infty} \frac{1}{n(\log n)^2}$.	06	CO3
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Q4	Solve the following	Marks	COs
a)	Evaluate $\int_0^{\frac{\pi}{2}} e^{-\sqrt{\tan \theta}} \cdot \sqrt[4]{\tan \theta} \cdot \sec^2 \theta \, d\theta$.	05	CO4
b)	Evaluate $\int_0^{\infty} 7^{-4x^2} \, dx$.	05	CO4
c)	Evaluate $\int_0^{\frac{\pi}{4}} \cos^3 2x \cdot \sin^4 4x \, dx$.	06	CO4

OR

c)	Evaluate $\int_3^7 \sqrt[4]{[(7-x)(x-3)]} \, dx$.	06	CO4
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Q5	Attempt Any Three from the following	Marks	COs
a)	Solve $\left(3x^2y + \frac{y}{x} \right) dx + (x^3 + \log x) dy = 0$.	06	CO5
b)	Solve $(x \cos x) \frac{dy}{dx} + y(x \sin x + \cos x) = 1$.	06	CO5
c)	In a simple closed circuit, the current 'i' at time 't' is given by, $L \frac{di}{dt} + Ri = E$. Find the current 'i', at time 't', given that, 'i=0' at 't=0' and 'L', 'R', 'E' are constants.	06	CO5
d)	Find orthogonal trajectory of $\frac{2a}{r} = 1 + \cos \theta$.	06	CO5

Q6

Solve the following

Marks

a) Prove that, $\log(1 + e^x) = \log 2 + \frac{1}{2}x + \frac{x^2}{8} - \frac{x^4}{192} + \dots$

05

b) Apply Taylor's theorem to find approximately the value of $f\left(\frac{11}{10}\right)$ where, $f(x) = x^3 + 3x^2 + 15x - 10$.

05

c) Evaluate $\lim_{x \rightarrow \infty} (\sinh x)^{\tanh x}$.

06

OR

c) Evaluate $\lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{1}{\log(x-2)} \right]$.

06
