



# Sanjay Ghodawat University, Kolhapur

2017-18

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

FY B. Tech.

School of Technology

Semester II

FYT 114

Complex Numbers and Calculus

Max Marks: 100

to Dec. 2018

End Semester Examination (ESE)

Time: 3 Hrs.

10:00 am to 1:00 pm

- Instructions for Students:**
- 1) Use of non-programmable calculator is allowed
  - 2) All questions are compulsory

Q1	Solve the following	Marks	COs
a)	Simplify, $\frac{[\cos 2\theta - i \sin 2\theta]^5 [\cos 3\theta + i \sin 3\theta]^6}{[\cos \theta - i \sin \theta]^8 [\cos 4\theta + i \sin 4\theta]^7}$ .	05	CO1
b)	Show that, $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$ .	05	CO1
c)	Solve, $x^4 - x^3 + x^2 - x + 1 = 0$ .	06	CO1
<b>OR</b>			
c)	Solve $x^{10} + 11x^5 + 10 = 0$ .	06	CO1
Q2	Attempt Any Three from the following	Marks	COs
a)	If $\sin(\alpha + i\beta) = x + iy$ , prove that,	06	CO2
	i) $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$ . ii) $\frac{x^2}{\sin^2\alpha} - \frac{y^2}{\cos^2\alpha} = 1$ .		
b)	Solve, $7\cosh x + 8\sinh x = 1$ , for real values of 'x'.	06	CO2
c)	Prove that $\tanh^{-1}z = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$ .	06	CO2
d)	Separate into real and imaginary parts $\tan^{-1}(\cos\theta + i\sin\theta)$ .	06	CO2

<b>Q3</b>	<b>Solve the following</b>	<b>Marks</b>	<b>COs</b>
a)	Using Comparison test, discuss the convergence of, $\frac{2.1^3+5}{4.1^5+1} + \frac{2.2^3+5}{4.2^5+1} + \dots$	05	CO3
b)	Using D'Alembert's ratio test, discuss the convergence of $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \dots, x > 0$	05	CO3
c)	Using Cauchy's $n^{\text{th}}$ root test, discuss the convergence of $\sum_1^\infty \left[ \left( \frac{n+1}{n} \right)^{n+1} - \left( \frac{n+1}{n} \right) \right]^{-n}$	06	CO3
<b>OR</b>			
c)	Examine the convergence of $\sum_1^\infty \frac{1}{n \log n}$ .	06	CO3
<b>Q4</b>	<b>Solve the following</b>	<b>Marks</b>	<b>COs</b>
a)	Evaluate $\int_0^1 (x \log x)^3 dx$ .	05	CO4
b)	Evaluate $\int_0^\infty x^{\frac{1}{4}} \cdot e^{-\sqrt{x}} dx$ .	05	CO4
c)	Evaluate $\int_0^\pi \sin^4 8x \cdot \cos^6 4x dx$ .	06	CO4
<b>OR</b>			
c)	Evaluate $\int_5^9 \sqrt[4]{[(9-x)(x-5)]} dx$ .	06	CO4
<b>Q5</b>	<b>Attempt Any Three from the following</b>	<b>Marks</b>	<b>COs</b>
a)	Solve $(1 + y^2 + 3x^2y)dx - (1 - 2xy - x^3)dy = 0$ .	06	CO5
b)	Solve $\frac{dy}{dx} + \frac{4xy}{x^2+1} = \frac{1}{(1+x^2)^3}$	06	CO5
c)	The charge 'q' on the plate of a condenser of capacity 'C', charged through a resistance 'R', by a steady voltage 'V' satisfies the differential equation $R \frac{dq}{dt} + \frac{q}{C} = V$ . If 'q=0' at 't=0' show that, $q = CV \left( 1 - e^{\frac{-t}{RC}} \right).$	06	CO5
d)	Find orthogonal trajectory of $r = a(1 + \cos \theta)$ .	06	CO5

Q6	Solve the Following	Marks	COs
a)	Prove that, $\sec^2 x = 1 + x^2 + \frac{2x^4}{3} + \dots$ .	05	CO6
b)	Expand $x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x-1)$ , and hence find $f\left(\frac{11}{10}\right)$ .	05	CO6
c)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{\tan x - \sin x}{\sin^3 x} \right)$	06	CO6
<b>OR</b>			
c)	Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} (\tan x)^{\cos x}$	06	CO6

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